2. A DIAGRAMMATIC APPROACH TO THE OPTIMAL LEVEL OF PUBLIC INPUTS

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Remarks


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2.1 Introduction

A key issue in the current debate over the provision of public goods has to do with their optimal levels. The controversy stems more from disagreement over what quantity of goods should be considered optimal than over the optimality rules deriving from their first order conditions. Wilson (1991), Chang (2000) and Gaube (2000, 2005, 2007), among others, highlight the importance of this issue, often using numerical examples (and counterexamples) to support their basic argument: namely, that since the optimal amount of public spending is inversely related to the welfare costs arising from distortionary taxation, the use of such taxation will open the door to an optimal spending scenario that falls short of its first-best level. Along these lines, Pigou (1947) argues that the total welfare cost of public spending must include not only its marginal production cost but also the deadweight loss generated by distortionary taxation. In his view, therefore, public expenditure should be greater in a lump-sum situation than in a second-best scenario. Yet this contention is not wholly indisputable, as Gaube (2000, 2005) has recently shown. Distributional concerns aside, Pigou’s intuition may be inaccurate with respect to his assumptions regarding the demand for taxed private commodities.

To date, these issues have received very little attention for the case of public inputs. However, like Feehan and Matsumoto (2002) we believe that the particular features of productive public spending merit closer study. In this chapter, we use a very simple model in which the optimal level of public inputs is discussed under two different tax settings. Our main result is that the first-best level of productivity-enhancing public expenditures is always higher than that of a second-best scenario. Our analysis is illustrated using a diagrammatic approach.

The rest of the chapter is structured as follows. Section 2 presents a simple theoretical framework. Section 3 shows the discussion and proof of the main result of the paper. Section 4 concludes.

2.2 The model

We consider an economy inhabited by a representative household with preferences over a single private commodity $x$ and leisure, given by the utility function $u(x,l)$, where $l$ is labor. The properties of this utility function are the standard ones to ensure a well-behaved function. The production possibility frontier is implicitly given by $f(x,l,g) = 0$, where $g$ is a public input. We normalize the public input price to be one. Total output $y$ can be costlessly used as $x$ or $g$. Consequently, if the resources devoted to the provision of $g$ are denoted by $R(g)$, then the amount
of private commodity available for consumption is \( x = y - R(g) \).

The representative consumer faces two different price vectors, depending on what tax system exists. When the first-best case is considered, prices are given by \((f_x, f_l)\). By contrast, when a second-best scenario is taken account, a distortion appears in the consumer prices, which we denote by \((q_x, q_l)\). In this sense, with taxes on consumption (labor), \( q_x > f_x \) \((q_l < f_l)\). In both cases, the household takes \( g \) as given.

Technology can be described by

\[
y = p(g) \times l,
\]

where \( p(g) \) is the average productivity of labor\(^1\). We assume that \( p'(g) > 0 \), that is, the productivity of labor is positively affected by the public input.

The production side of the economy can be simplified by using results from the envelope theorem. Particularly, for all values of \( g \), a production possibility frontier can be defined in the space of \( x \) and \( l \).

Definition 1: Given the equation \( f(x, l, g) = 0 \), the production possibility frontier is defined in the space of \( x \) and \( l \) as the envelope of a set of curves solving the above equation system for all values of public input \( g \):

\[
\begin{align*}
f(x, l, g) &= 0 \\
f_g(x, l, g) &= 0
\end{align*}
\]

Figure 2.1 shows how the production possibility frontier \((\text{broader line in red})\) is drawn on the basis of Definition 1. For all values of \( g \), and consequently \( R(g) \), the production function (2.1) gives a pair of values \((x, l)\) which maximize the total output \( y \). When the values of \( g \) are modified, different combinations of \( p(g) \) and \( R(g) \) give different points of the production possibilities frontier. This curve is a convex function as long as \( p'(g) \) and \( R'(g) \) are positive.

2.3 The comparison of optimal levels of provision

The intuition behind an optimal allocation is that the production possibility frontier and the highest possible indifference curve are tangential to each other. Formally, we establish that the first-best allocation \((x^*, l^*, g^*)\) is a tangential point between the production possibility frontier

\[\]
and the indifference curve $u^*$:

$$\frac{f_l}{f_x} = MRS_l^x. \quad (2.3)$$

Point A in Figure 2 represents the first-best allocation. On the other hand, the second-best bundle $(x^{**}, l^{**}, g^{**})$ is that on the production possibility frontier in which the slope of consumer budget constraint equals the marginal rates of substitution of the as highest possible indifference curve:

$$\frac{g_l}{q_x} = MRS_l^x. \quad (2.4)$$

Obviously, $\frac{g_l}{q_x} < \frac{f_l}{f_x}$ at the second-best optimum, and this is the key point for demonstrating the main result of this paper:

Proposition 1: In a single private commodity economy with a representative household, the optimal level of public inputs in a second-best scenario is always lower than that of the first-best equilibrium.

Proof. Assume that the second-best level is higher than the first-best level: $g^{**} > g^*$. Since the production possibility frontier is convex, the consumer budget constraint in the second-best scenario cuts across the first-best indifference curve $u^*$. This implies that the indifference curve $u^{**}$, which is tangent to the consumer budget constraint (blue line in Figure 2.2) at the second-best allocation (B in Figure 2.2), must intersect indifference curve $u^*$ of the first-best case. This is a contradiction. ■

In line with this inconsistency, note that a second-best level of public inputs which is higher than the first-best level would imply a higher consumption and labor supply than with non-distorting taxes, and this is not possible in this framework.

From a complementary point of view, it can be easily proved that a second-best level of public inputs lower than the first-best level is consistent with rationality. Indeed, on the basis of Figure 2.3, the second-best allocation C does not lead to two indifference curves which intersect each other. Now the consumer budget constraint (blue line in Figure 2.3) tangent to the curve $u^{**}$ does not cut across the first-best indifference curve, although the slope of the production possibility frontier at bundle C continues to be higher than the slope of the budget constraint with distorting taxation.
2.4 Conclusion

This short chapter deals with the optimal level of public inputs regarding whether the taxes are distorting or not. Previous papers have focussed on this issue in the case of public goods but nothing had said on productivity-enhancing public spending. The main difference in the analysis of the two types of government expenditures lies in the relevant function to be studied. While the public goods are arguments in utility functions, but public inputs are arguments in production functions.

The main result of this paper states that the optimal level of public inputs in a first-best scenario is always higher than in the second-best case. Our paper follows a simple but very intuitive approach to demonstrate that financing public inputs with distortionary taxation leads to lower levels of public spending. In a sense, in line with Pigou’s intuition, efficiency costs of consumption and labor taxes have to be taken account not only to design the optimal rule for the provision of public spending but also to define the optimal size of government.

Our result could be expanded along several lines. Firstly, public input can be subject to some degree of congestion. Hence, taxes on consumption or other private production factors could imply an efficiency improvement. Therefore, distortionary taxation would increase the effective amount of public inputs, given the nominal level provided by the government. Secondly, the likelihood of a level reversal could be higher in a multi-private commodity economy where a more complex structure of scenarios might arise. In this sense, and depending on whether the taxed goods and the public input are complement or not, a positive feedback effect might increase tax revenues and, hence, the level of public spending in the presence of distortionary taxation.
2.5 References


Notes

1With a Cobb-Douglas production function, it can be easily shown that no particular assumptions on the returns to scale are necessary.
2.7 Figures

Fig. 2.1: Production possibility frontier (PPF)
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Fig. 2.3: SB level lower than the FB level

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